Star Formation 2020

Q&A 26.05.2020

Giant Molecular Clouds

H_2 formation in the early universe

Star formation via cloud collapse depends on the balance between gravitational attraction and thermal gas pressure. The lower the temperature the fewer mass is required to allow the cloud to collapse. One way to cool the cloud is by molecular line emission. Molecules like, H_2, CO, and H_2O contribute strongly to the overall energy balance of a molecular cloud. However, dust and elements heavier than deuterium are only formed after the first generation of stars were borne. This leads to the question how star formation via cloud collapse happened without any dust to form H_2 on (H_2 is considered the only coolant below 10^4 K in low metallicity environments).

A much more inefficient formation route for H_2, in the absence of dust surfaces, is the association of H atoms with H^- ions

$$\mathrm{H} + H^- \longrightarrow H_2 + e^-$$

The H_2 formation rate via this reaction is

$$R_g = k_2\, n_H\, n_{H^-} \tag{1}$$

To calculate R_g we need to know the densities of H and H^-, The equilibrium H^- density is driven by the following reactions:

1)	$\mathrm{H} + e^- \longrightarrow H^- + h\nu$		$k_1 = 1.4 \times 10^{-18}\, T^{0.928}\, \exp(-T/16200)$
2)	$H^- + \mathrm{H} \longrightarrow H_2 + e^-$		$k_2 = 1.3 \times 10^{-9}$
3)	$H^- + h\nu \longrightarrow \mathrm{H} + e^-$		$k_3 = 2.4 \times 10^{-7}\, I_{\mathrm{FUV}}\, \exp(-0.5\, A_V)$
			$k_3 = 0.11\, T_{\mathrm{rad}}^{2.13}\, \exp(-\frac{8823}{T_{\mathrm{rad}}})$
4)	$H^+ + H^- \longrightarrow \mathrm{H} + \mathrm{H}$		$k_4 = 7 \times 10^{-7}\, T^{-0.5}$

H^- density

Show that the equilibrium density of H^- can be expressed as:

$$n_{H^-} = \frac{k_1\, n_e\, n_H}{k_2\, n_H + k_3 + k_4\, n_{H^+}} \tag{2}$$

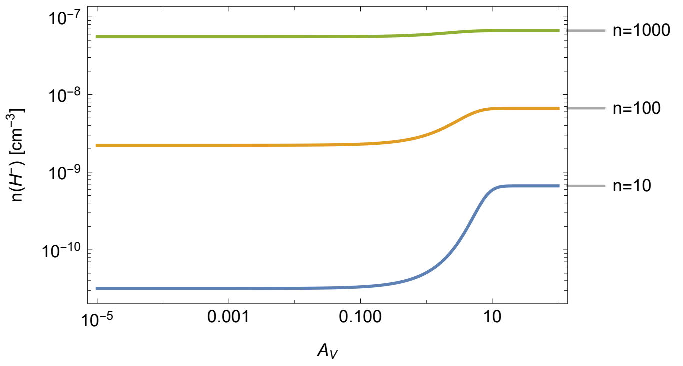
Reaction 1) is a formation reaction of H^- with the rate: $k_1\, n_H\, n_e$
Reactions 2) - 4) are destruction reactions

Balancing formation and and destruction reactions gives $k_1\, n_e\, n_H = n_{H^-}(k_2\, n_H + k_3 + k_4\, n_{H^+})$. Rearranging gives Eq. 2)

Plot n_{H^-} as a function of A_V assuming that $n_e = n_{H^+} = n_H\, 10^{-4}$ and $T = 1000$ K for the $n_H = 10$, 100, and 1000 cm^{-3}.

In[14]:=
```
nHm[nH_, ne_, T_, IFUV_, AV_] := (1.4 × 10^-18 T^0.928 Exp[-T / 16 200] ne nH) /
  (1.2 × 10^-9 nH + 2.4 × 10^-7 IFUV Exp[-0.5 AV] + 7 × 10^-7 T^-0.5 ne)
```

In[45]:=
```
LogLogPlot[{nHm[10, 10 * 10^-4, 1000, 1, AV],
  nHm[100, 100 * 10^-4, 1000, 1, AV], nHm[1000, 1000 * 10^-4, 1000, 1, AV]},
 {AV, 10^-5, 100}, PlotLabels → {"n=10", "n=100", "n=1000"},
 Frame → True, FrameLabel → {"AV", "n(H⁻) [cm^-3]"}]
```

Out[45]=


Alternatively, we can formulate this as a function of redshift:

In[49]:=
```
TCBR[z_] := 2.73 * (1 + z)
(*Tgas[z_]:=1*(1+z)^2*)
dens[z_, Ω_ : 0.1, H0_ : 70] := 212.5 ((1 + z)/1000)^3 (Ω/0.1)^4 (H0/50)^8
nHm[z_, ξ_] := (1.4 × 10^-18 TCBR[z]^0.928 Exp[-TCBR[z] / 16 200] ξ dens[z]^2) /
  (1.2 × 10^-9 dens[z] + 0.11 TCBR[z]^2.13 Exp[-8823/TCBR[z]] + 7 × 10^-7 TCBR[z]^-0.5 ξ dens[z])
```

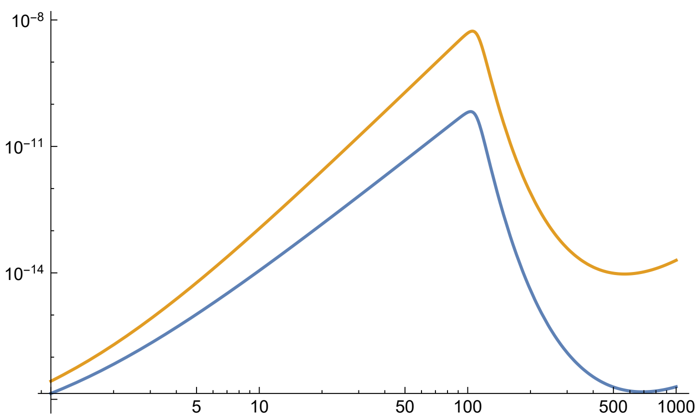
In[54]:=
```
LogLogPlot[{nHm[z, 10^-4], nHm[z, 10^-4 (1 + z)]}, {z, 1, 1000}]
```

General: Exp[-1615.82] is too small to represent as a normalized machine number; precision may be lost.

General: Exp[-1615.82] is too small to represent as a normalized machine number; precision may be lost.

General: Exp[-1615.82] is too small to represent as a normalized machine number; precision may be lost.

General: Further output of General::munfl will be suppressed during this calculation.

Out[54]=


H_2 formation rate

For the remainder of this problem set we ignore reaction 3) and 4). Give an expression for the reaction rate of H_2

Plot R_g as a function of T for n_H = 100 and ξ_e=3 × 10^{-4}.

Solution

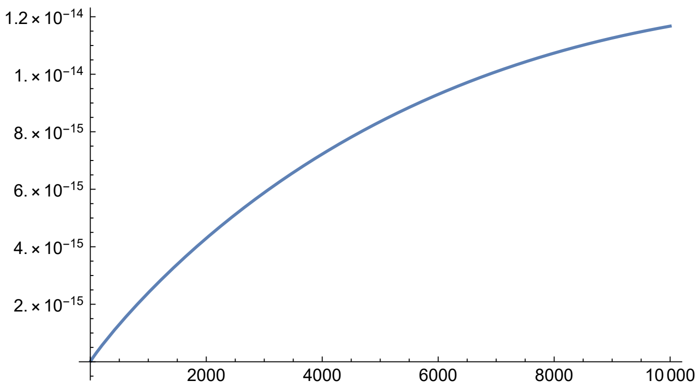
$$n_{H^-} \approx \frac{k_1\, n_e\, n_H}{k_2\, n_H} \tag{3}$$

$$R_g = k_2\, n_{H^-}\, n_H = k_2 \frac{k_1\, n_e\, n_H}{k_2\, n_H} n_H = k_1\, n_e\, n_H \tag{4}$$

or expressed using the ionization fraction $\xi_e = \frac{n_e}{n_H}$

$$R_g = k_1\, \xi_e\, n_H^2 \tag{5}$$

In[55]:=
```
Rg[nH_, ξe_, T_] := 1.4 × 10^-18 T^0.928 Exp[-T / 16 200] ξe nH^2
Plot[{Rg[100, 3 × 10^-4, T]}, {T, 10, 10 000}]
```

Out[56]=


Jeans Mass

The Jeans mass can be expressed as

$$M_J \approx 45\, M_\odot\, T^{3/2}\, n_{cm^{-3}}^{-1/2} \tag{6}$$

Compare the smallest possible mass capable of collapse at the time of decoupling (T≈3000K, n≈ 6000 cm^{-3}) and today (T≈50K, n≈1000 cm^{-3}).

Solution

```
Mj[T_, n_] := 45 T^3/2 n^-1/2

Mj[3000, 6000.]
Mj[50, 1000.]
95 459.4

503.115
```

Dust vs gas formation route

In the presence of dust, H_2 is formed on the surface of dust grains. The reaction rate can be written as

$$S(T_g, T_d) = \frac{1}{0.04\sqrt{T_d + T_g} + \frac{8 T_g^2}{10^6} + \frac{2 T_g}{10^3} + 1}$$

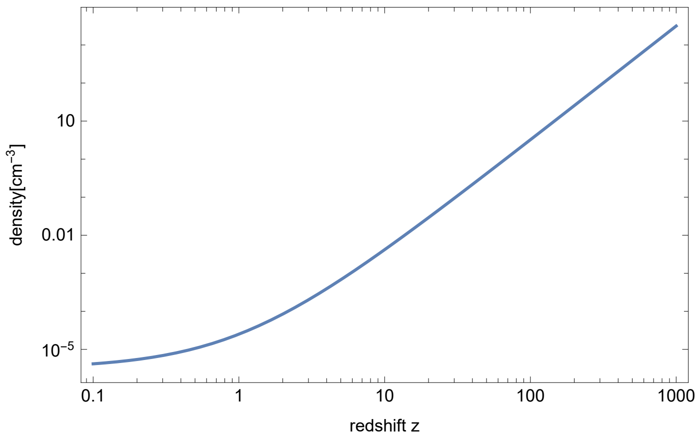
$$f_a(T_d) = \frac{1}{10^4 \exp(-\frac{600}{T_d}) + 1}$$

$$R_d = \frac{3\, n\, n_H \sqrt{T_g}\, f_a(T_d)\, S(T_g, T_d)}{10^{18}}$$

with the gas and dust temperatures T_g and T_d and the total density n. Assume $n = n_H$=1 cm^{-3} and T_d = 100 K and ξ_e=3 × 10^{-4} and find a temperature for which $R_g = R_d$. What if n = 100 cm^{-3}? Plot both functions as a function of T.

Solution

```
In[40]:= Tgas[1.]
Out[40]= 0.012

In[41]:= TCBR[z_] := 2.73 * (1 + z)
         Tgas[z_] := 1 * (1 + z)^2
         dens[z_, Ω_ : 0.1, H0_ : 70] := 212.5 ((1 + z)/1000)^3 (Ω/0.1)^4 (H0/50)^8

In[44]:= LogLogPlot[dens[z], {z, 0.1, 1000},
          Frame → True, FrameLabel → {"redshift z", "density[cm^-3]"}]
Out[44]=
```

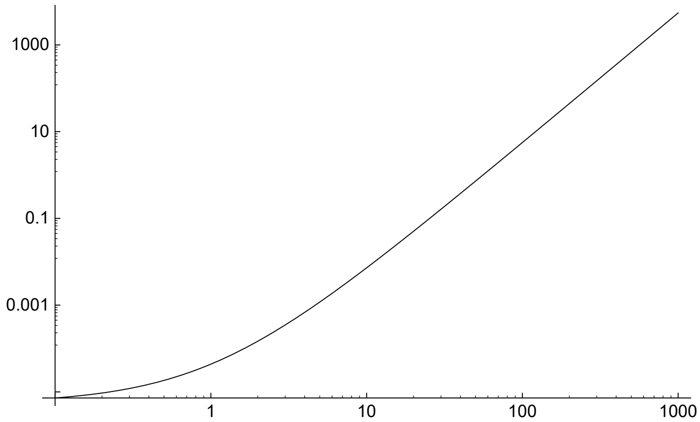



```
In[9]:= Rg[nH_, ξe_, T_] := 1.4 × 10^-18 T^0.928 Exp[-T / 16200] ξe nH^2

In[10]:= S[Tg_, Td_] := (1 + 0.04 (Tg + Td)^(1/2) + 2 × 10^-3 Tg + 8 × 10^-6 Tg^2)^-1
         fa[Td_] := 1/(1 + 10^4 Exp[-600 / Td])
         R[n_, nH_, Tg_, Td_] := 3 × 10^-18 fa[Td] × S[Tg, Td] √Tg n nH

In[∘]:= FindRoot[Rg[1, 3 × 10^-4, T] == R[1, 1, T, 100], {T, 1000}]
Out[∘]= {T → 1121.24}
```

In[22]:=
```
FindRoot[Rg[100, 3 × 10^-4, T] == R[100, 100, T, 100], {T, 1000}]
```

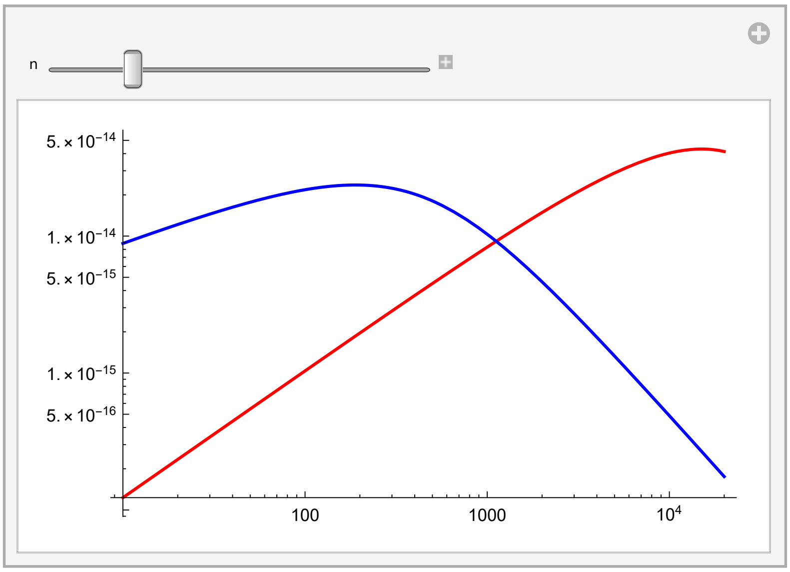
Out[22]=
```
{T → 1121.24}
```

```
LogLogPlot[nH[z], {z, 0.1, 1000}]
```



In[17]:=
```
Manipulate[LogLogPlot[{Rg[n, 3 × 10^-4, T], R[n, n, T, 100]}, {T, 10, 20000},
  PlotStyle → {Red, Blue}], {{n, 100}, 0, 1000}, ControlPlacement → Top]
```

Out[17]=



Magnetic Support of Clouds

Consider a spherical cloud of gas of initial mass M, radius R, and velocity dispersion σ, threaded by a magnetic field of strength B. In class we showed that there exists a critical magnetic flux M_ϕ such that, if the cloud's mass $M < M_\phi$, the cloud is unable to collapse.

a) Show that the the cloud's Alfvén Mach number $\mathcal{M}_A$ depends only on its virial ratio α_{vir} and on $\mu_\phi \equiv M/M_\phi$ alone. Do not worry about constants of order unity.

The virial ratio is (omitting constant factors of order unity)

$$\alpha_{vir} \sim \frac{\sigma^2 R}{G M}$$

The Alfvén Mach number is the ratio of the velocity dispersion to the Alfvén speed

$$v_A \sim \frac{B}{\sqrt{\rho}} \sim \frac{B\,R^{3/2}}{M^{1/2}}$$

thus

$$\mathcal{M}_A \sim \frac{\sigma\,M^{1/2}}{B\,R^{3/2}}$$

To rewrite this in terms of M_ϕ, we can eliminate B from this expression by writing

$$B \sim \frac{M_\phi\,G^{1/2}}{R^2}$$

giving

$$\mathcal{M}_A \sim \frac{\sigma}{M_\phi}\sqrt{\frac{M\,R}{G}}$$

Similarly, we can eliminate σ using the definition of the virial ratio:

$$\sigma \sim \sqrt{\alpha_{vir}\,\frac{G\,M}{R}}$$

and substituting this in gives

$$\mathcal{M}_A \sim \alpha_{vir}^{1/2}\,\mu_\phi$$

b) Your result from the previous part should demonstrate that, if any two of the dimensionless quantities μ_ϕ, α_{vir}, and $\mathcal{M}_A$ are of order unity, then the third quantity must be as well. Give an intuitive explanation of this result in terms of the ratios of energies (or energy densities) in the cloud.

The expression derived in part (a) does indeed show that, if any of two of the three quantities

$\mathcal{M}_A$, α_{vir}, and μ_ϕ

are of order unity, the third one must be as well. Intuitively, this is because the various quantities are measures of energy ratios. Roughly speaking,

- $\mathcal{M}_A^2$ measures the ratio of kinetic (including thermal) energy to magnetic energy;
- α_{vir} measures the ratio of kinetic to gravitational energy; and
- μ_ϕ^2 represents the ratio of gravitational to magnetic energy.

If any two of these are of order unity, then this implies that gravitational, kinetic, and magnetic energies are all of the same order. However, this in turn implies that the third dimensionless ratio should also be of order unity as well.

For example, if $\mathcal{M}_A \sim \alpha_{vir} \sim 1$, then this implies that kinetic energy is comparable to magnetic energy, and kinetic energy is also comparable to gravitational energy. In turn, this means that gravitational and magnetic energy are comparable, in which case $\mu_\phi \sim 1$

c) Magnetized turbulence naturally produces Alfvén Mach numbers $\mathcal{M}_A \sim 1$. Using this fact plus your responses to the previous parts, explain why this makes it difficult to determine observationally whether clouds are supported by turbulence or magnetic fields.

If we have a cloud that is supported, it must have $\alpha_{\text{vir}} \sim 1$. However, if the cloud is turbulent then it will naturally also go to $\mathcal{M}_A \sim 1$. This means that we are likely to measure $\mu_\phi \sim 1$ even if the cloud is magnetically supercritical and not supported by its magnetic field.

We would only ever expect to get $\mu_\phi >> 1$, indicating a lack of magnetic support, if the cloud were either non-virialized ($\alpha_{\text{vir}} >> 1$ or $<< 1$) or non-turbulent.